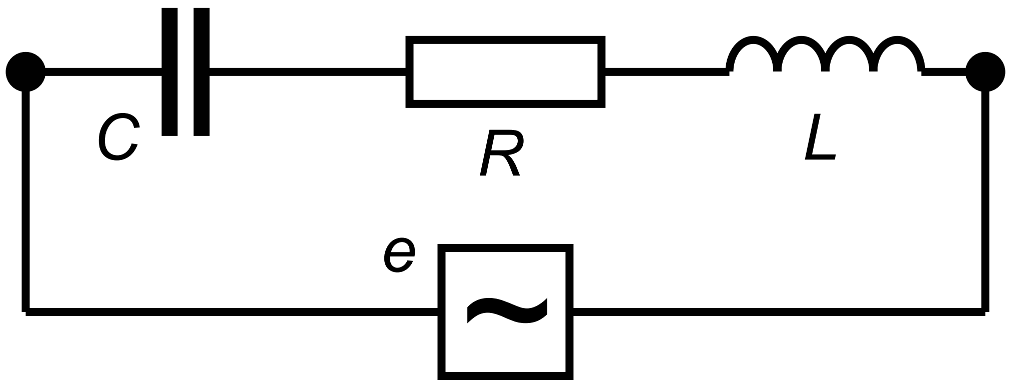
# 5. PHASORS

**Main things to learn**

- Phasor representation
- Phasor diagrams for resistive, capacitive and inductive circuits
- Addition and subtraction of phasors
- Application to circuits in-series




$$u = u_C + u_R + u_L =$$
$$= I_m Z \sin(\omega t + \varphi) \quad \text{where}$$

$$\varphi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

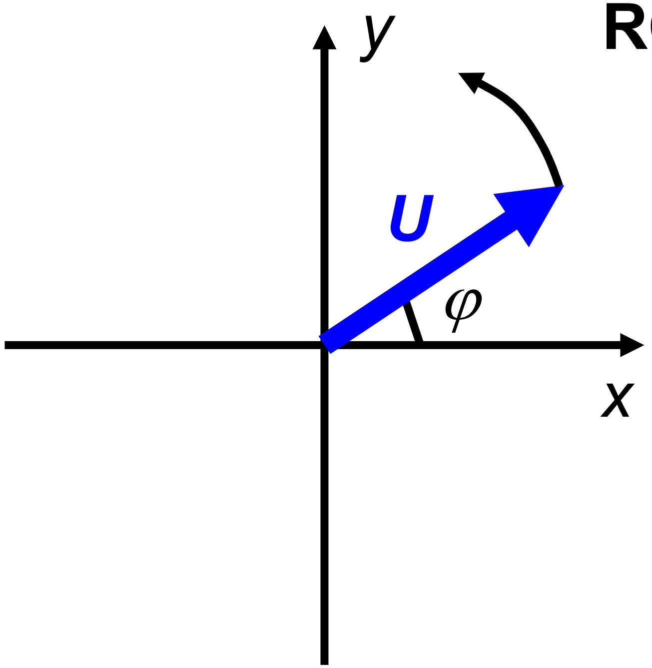
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

**Aim:**

To develop a **practical** and **illustrative** method

for the analysis of such and more complicated circuits

# ROTATIONAL MOTION




An arrow (a vector) of length $U$ starts from the beginning of the $X$-$Y$ coordinate plane.

$\varphi$ - angle between the arrow and the $X$-axis

Coordinates of the end of the arrow are:

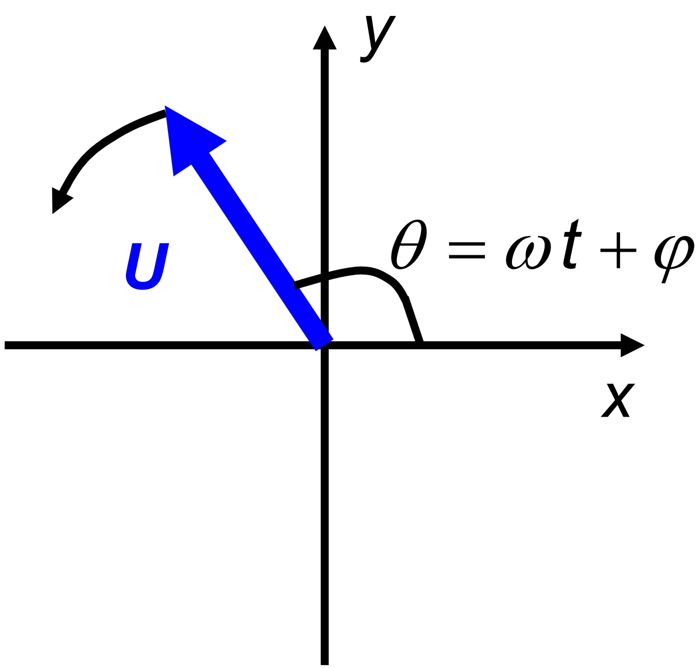
$$x = U\cos\varphi \quad ; \quad y = U\sin\varphi$$

If the arrow rotates **anti-clockwise**

with a constant angular speed $\omega$,

the angle $\theta$ between the arrow and the $X$-axis is

$$\theta = \omega t + \varphi$$




Therefore, the coordinates of the end of the arrow are:

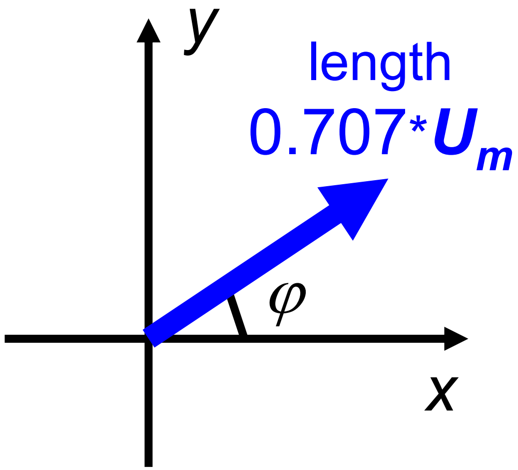
$$x = U\cos(\omega t + \varphi) \ ; \quad y = U\sin(\omega t + \varphi)$$

These equations are identical to the equations for the alternating current

$\varphi$ is the **initial phase** at $t$ = 0

# PHASOR REPRESENTATION

Basis: similarity between rotational motion and periodic processes




$$u = U_m \sin(\omega t + \varphi)$$

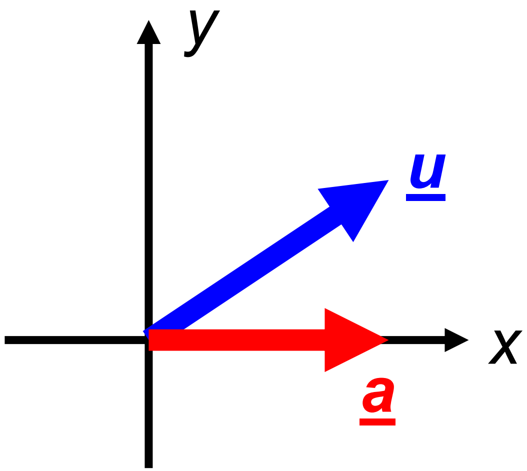
**Phasor** for the quantity $u$ is a **vector** which has

- Length $0.707 {}^* U_m$
- Angle $\varphi$ between the vector and the $X$-axis

We **do not** represent the term $\omega t$ because it is the same for all quantities.

**Of importance is the phase difference**

between $u$ and a **reference** quantity $a$ for which the initial phase $\varphi = 0$

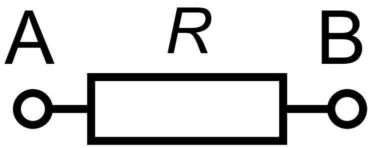



$$a = A_m \sin \omega t$$

**Notation for phasors**

- In textbooks - typically bold like ***a***
- In these handouts - underlined like ***a***
- **Length of the phasor is the r.m.s. value**, not the amplitude - this is why $0.707 {}^* U_m$

# PHASOR DIAGRAM FOR A RESISTOR

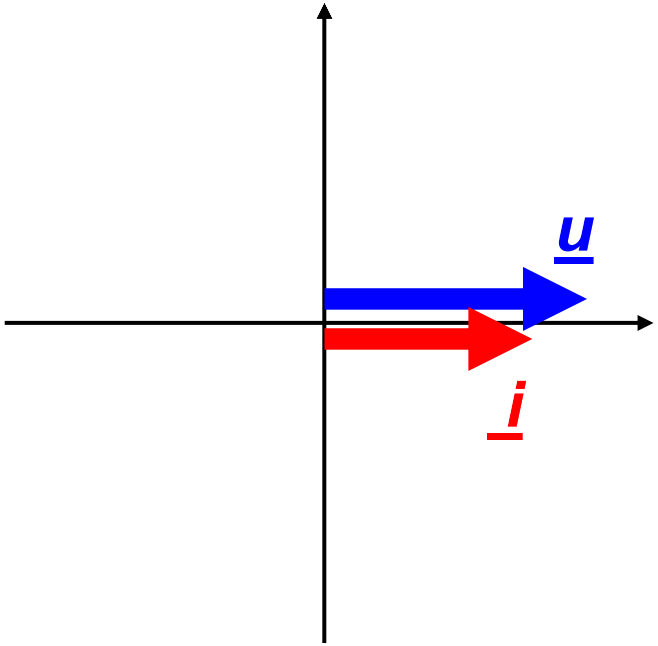



$$i = I_m \sin\omega t$$

$$u_{AB} = I_m R \sin\omega t$$

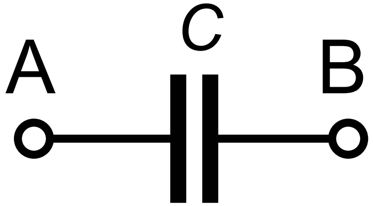
Phase difference between voltage and current is zero

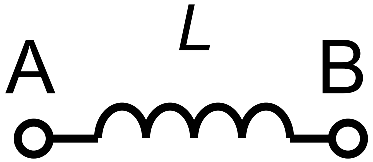
**Current and voltage are in-phase**




**Current in the resistor is taken as reference**

# PHASOR DIAGRAMS FOR A CAPACITOR AND AN INDUCTOR




$$i = I_m \sin\omega t$$

$$u_{AB} = I_m X_C \sin\left(\omega t - \pi/2\right)$$

$$i = I_m \sin\omega t$$

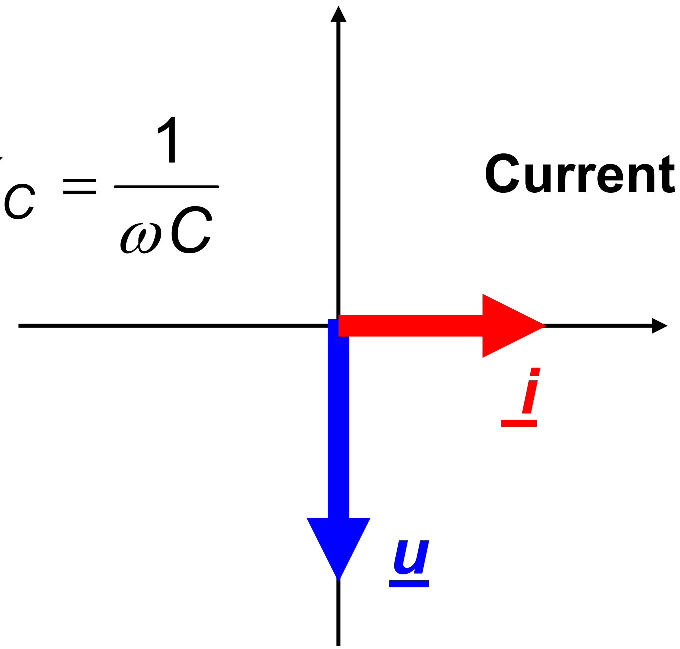
$$u_{AB} = I_m X_L \sin\left(\omega t + \pi/2\right)$$

Phase difference between voltage and current is $\pi/2$ (quarter-cycle )

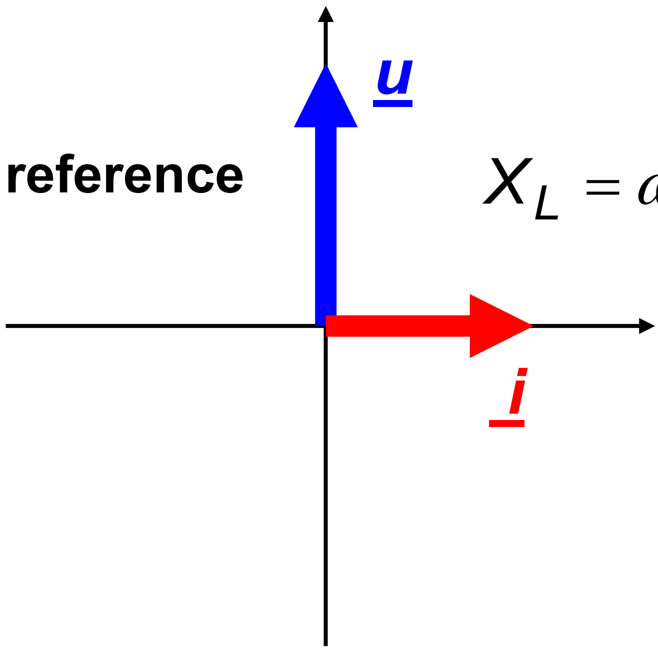
**Voltage lags current**

**Voltage leads current**

$$X_C = \frac{1}{\omega C}$$




**Current is taken as reference**




$$X_L = \omega L$$

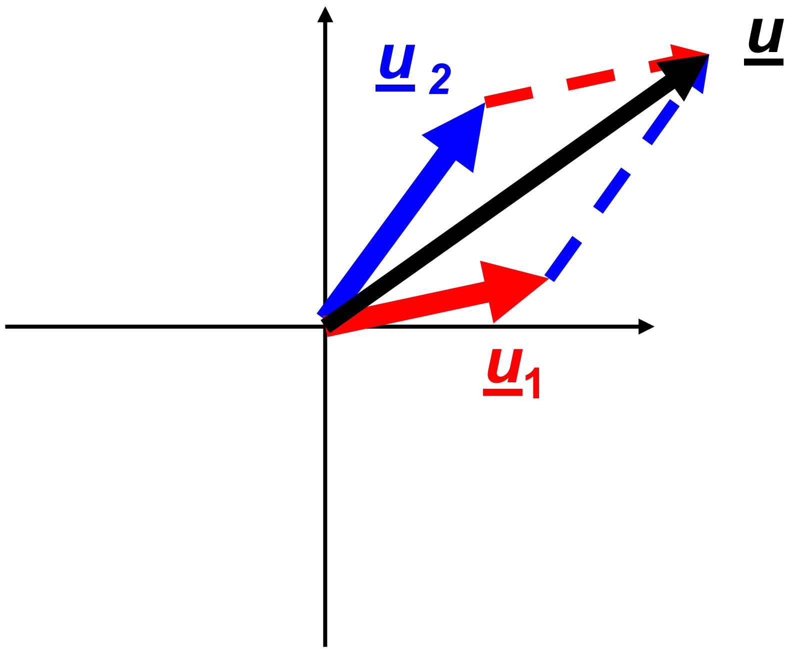
# ADDITION OF PHASORS

Why do we need phasors?

We want to add two alternating voltages

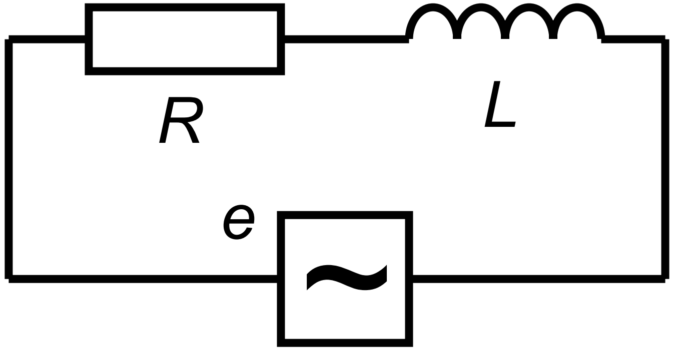
$$u_1 = U_{1m} \sin(\omega t + \varphi_1)$$
$$u_2 = U_{2m} \sin(\omega t + \varphi_2)$$

Their sum $u = u_1 + u_2$ can be represented by a phasor $\underline{\boldsymbol{u}}$ which is a vector (phasor) sum of the phasors $\underline{\boldsymbol{u}}_1$ and $\underline{\boldsymbol{u}}_2$




The amplitude $U$ and the phase $\varphi$ for the phasor $\underline{\boldsymbol{u}}$ can be found geometrically

# RESISTANCE AND INDUCTANCE IN SERIES

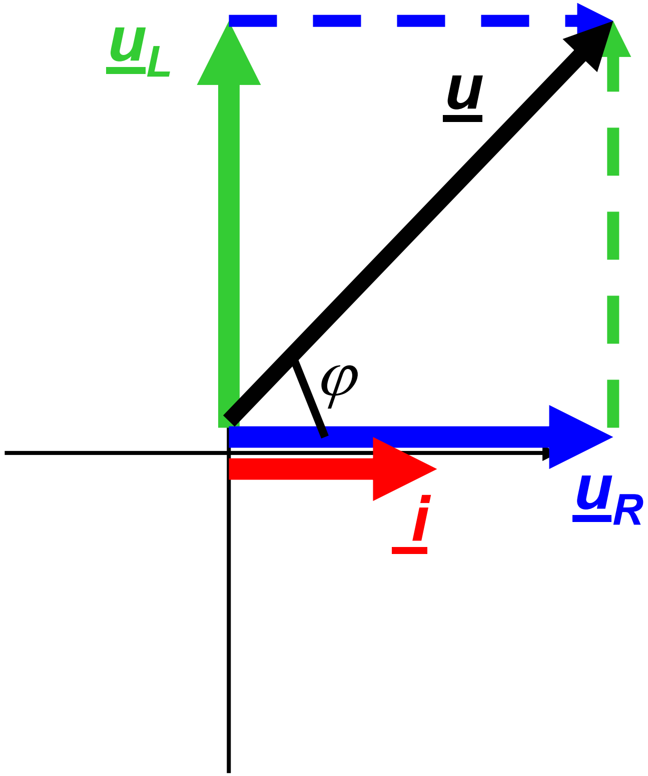



$U_R = I\,R$ : $U_R$ in-phase with $I$

$U_L = I\,X_L$ : $U_L$ leads $I$ by $\pi/2$

$$X_L = \omega L$$

**Current is taken as reference because it is the same for both elements**




The total voltage $u$ leads current $i$

**Phase:**

$$\varphi = \tan^{-1}\frac{U_L}{U_R} = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{\text{reactance}}{\text{resistance}}\right)$$

**Amplitude:**

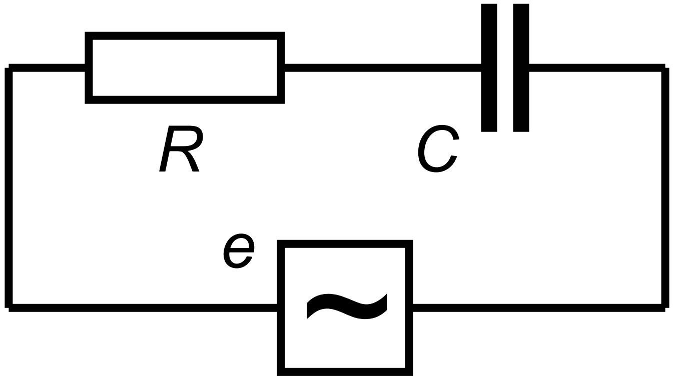
$$U = \sqrt{U_L{}^2 + U_R{}^2} = I\sqrt{X_L{}^2 + R^2} = IZ$$

**Impedance:**

$$Z = \sqrt{X_L{}^2 + R^2} = \sqrt{(\omega L)^2 + R^2}$$

$$\text{Impedance} = \sqrt{\text{Resistance}^2 + \text{Reactance}^2}$$

# RESISTANCE AND CAPACITANCE IN SERIES

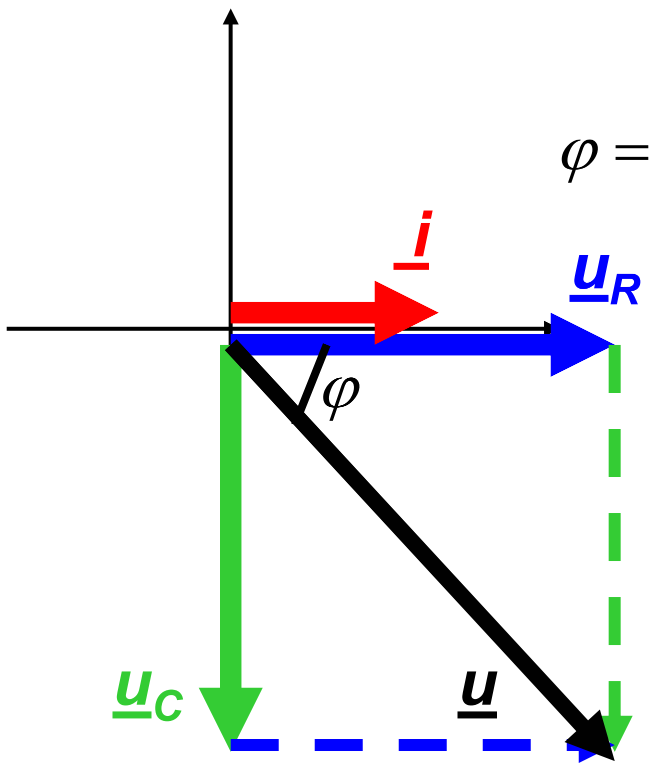



$U_R = I\,R$ : $U_R$ in-phase with $I$

$U_C = I\,X_C$ : $U_L$ lags $I$ by $\pi/2$

$$X_C = \frac{1}{\omega C}$$

**Current is taken as reference because it is the same for both elements**




The total voltage $u$ lags current $i$

**Phase:**

$$\varphi = -\tan^{-1}\frac{U_C}{U_R} = -\tan^{-1}\frac{X_C}{R} = -\tan^{-1}\left(\frac{\text{reactance}}{\text{resistance}}\right)$$

**Amplitude:**

$$U = \sqrt{U_C^{\,2} + U_R^{\,2}} = I\sqrt{X_C^{\,2} + R^2} = IZ$$

**Impedance:**

$$Z = \sqrt{X_C^{\,2} + R^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$$

$$\text{Impedance} = \sqrt{\text{Resistance}^2 + \text{Reactance}^2}$$

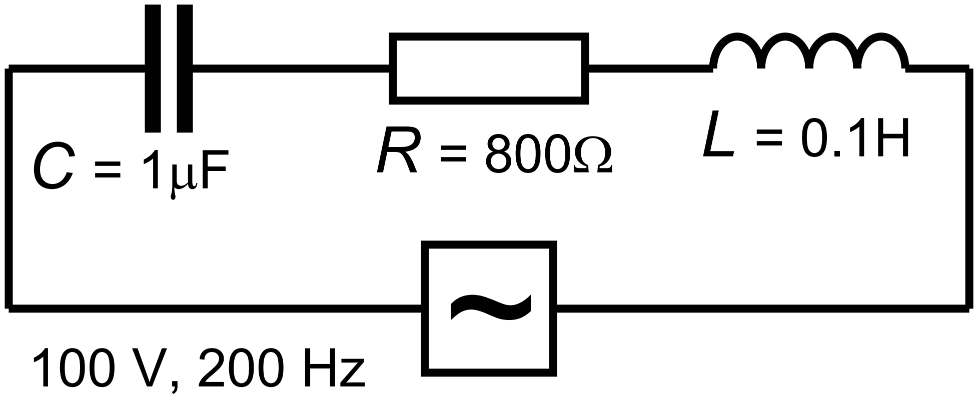
# HOW TO SOLVE PROBLEMS: *LCR* CIRCUIT

A resistor of resistance $R = 800\ \Omega$, a capacitor of capacitance $C = 1\ \mu\text{F}$ and a coil of inductance $L = 0.1$ H are connected in series to a voltage source 100 V, 200 Hz. Determine the impedance of the circuit, the current and the phase difference between the voltage and the current.

1. Draw a circuit diagram
2. Insert all the known quantities
3. Determine the reactances
4. Choose reference quantity
5. Plot the phasor diagram (approximately to scale)
6. Determine the impedance
7. Determine the current
8. Determine the phase

# PROBLEM SOLUTION

**1-2)**

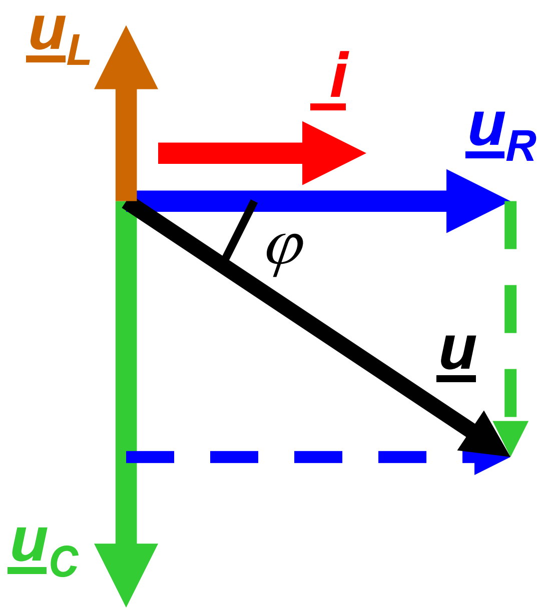



**3)**

$$X_L = 2\pi \times 200 \times 0.1 = 126\ \Omega$$

$$X_C = 1/(2\pi \times 200 \times 10^{-6}) = 796\ \Omega$$

**4-5)**




**6)** $Z = \sqrt{(796 - 126)^2 + 800^2} = 1044\Omega$

**7)** $I = 100/1044 = 96$ mA

**8)** $\varphi = -\tan^{-1}\left(\frac{796 - 126}{800}\right) = -40^{o}$

Voltage lags current by 40°

Current leads voltage by 40°