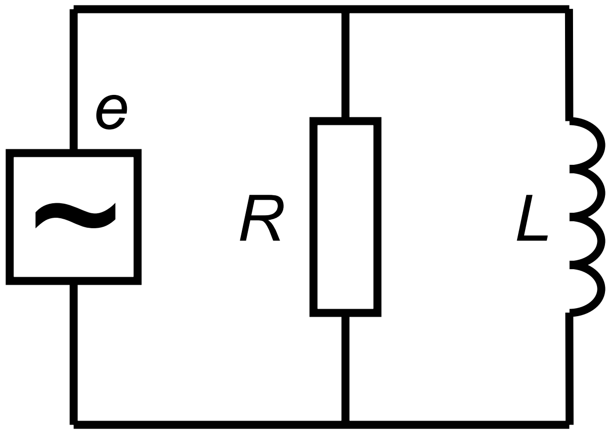
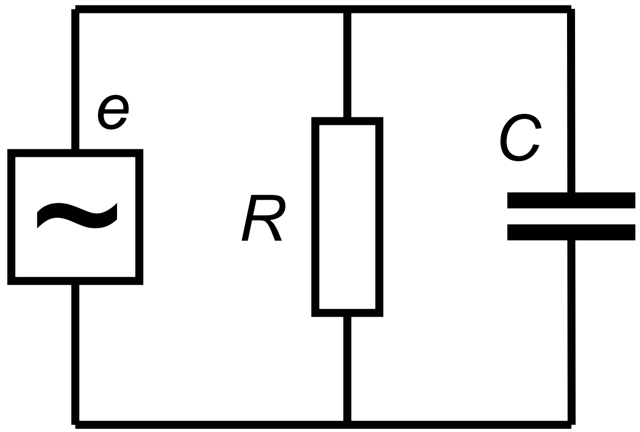
6. PARALLEL A.C. CIRCUITS

Main things to learn

- Analysis of parallel a.c. circuits
- Active and reactive current
- Admittance and susceptance
- Power factor

Common way of connecting loads to the power supply is **in parallel**





CR or *LR* circuits in parallel. How to analyse them using phasor diagrams?

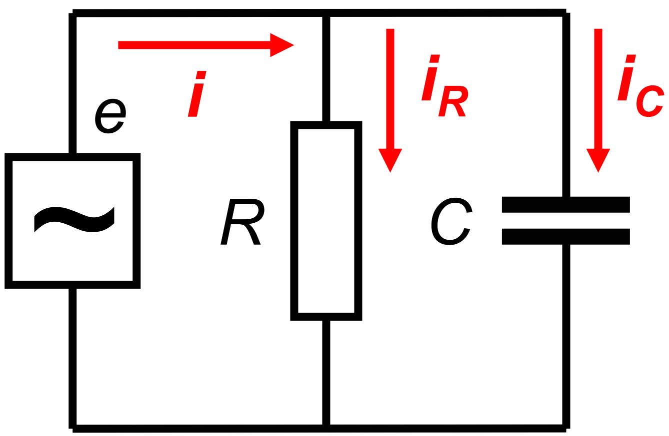
For series circuits, it was reasonable to take **current as a reference quantity**

However, in parallel circuits the voltage is the same across each element

Therefore, voltage is the most reasonable choice for reference

It is common to use the voltage from the power supply as reference

PHASOR DIAGRAM FOR A *CR* CIRCUIT IN PARALLEL



$$I_R = U/R \quad : \quad i_R \text{ in-phase with } u$$

$$I_C = U/X_C \quad : \quad i_C \text{ leads } u \text{ by } \pi/2 \ , \ X_C = 1/\omega C$$

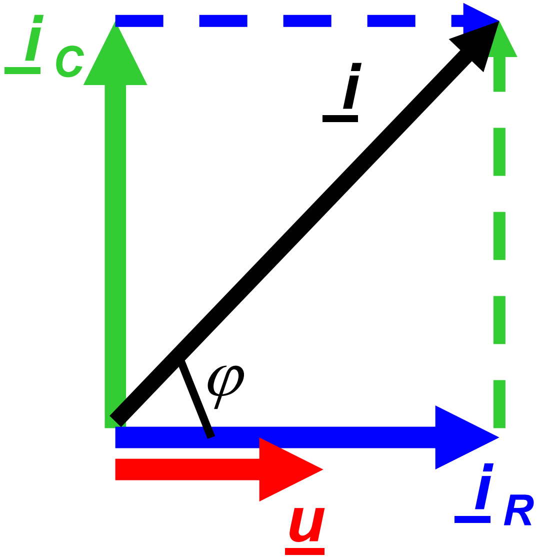
Phase:

The total current *i* leads voltage *u*
(like for a series *CR* circuit)

$$\varphi = \tan^{-1}\frac{I_C}{I_R} = \tan^{-1}\frac{R}{X_C} = \tan^{-1}\left(\frac{\text{resistance}}{\text{reactance}}\right)$$

$$\underline{i} = \underline{i}_R + \underline{i}_C$$

Voltage *u* - reference



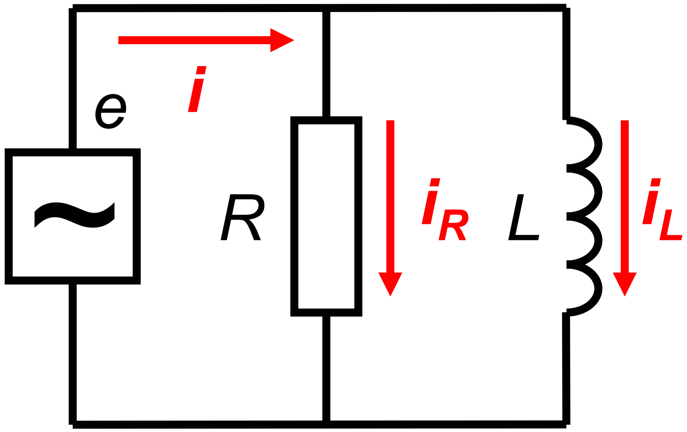
Amplitude:

$$I = \sqrt{I_C^{\,2} + I_R^{\,2}} = U\sqrt{\frac{1}{X_C^{\,2}} + \frac{1}{R^2}} = \frac{U}{Z}$$

Impedance:

$$\frac{1}{Z} = \sqrt{\frac{1}{X_C^{\,2}} + \frac{1}{R^2}} = \sqrt{(\omega C)^2 + \frac{1}{R^2}}$$

PHASOR DIAGRAM FOR AN *LR* CIRCUIT IN PARALLEL

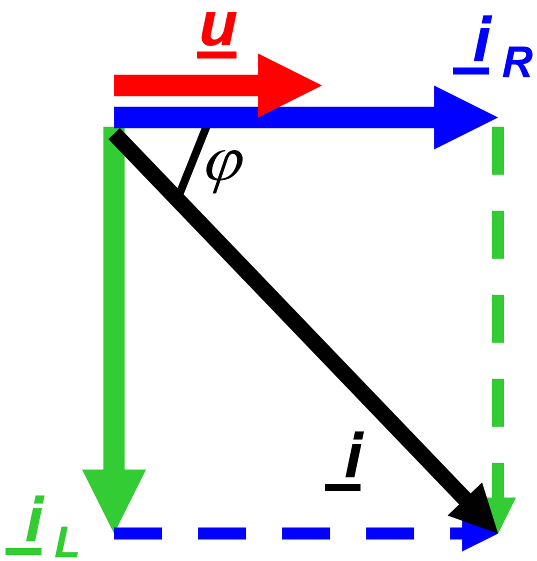


$$I_R = U/R \quad : \quad i_R \text{ in - phase with } u$$

$$I_C = U/X_L \quad : \; i_L \text{ lags } u \text{ by } \pi/2 \;,\; X_L = \omega L$$

$$\underline{i} = \underline{i}_R + \underline{i}_L$$

Voltage *u* - reference



Phase:

The total current *i* lags voltage *u*

(like for a series L*R* circuit)

$$\varphi = -\tan^{-1}\frac{I_L}{I_R} = -\tan^{-1}\frac{R}{X_L} = -\tan^{-1}\left(\frac{\text{resistance}}{\text{reactance}}\right)$$

Amplitude:

$$I = \sqrt{I_L{}^2 + I_R{}^2} = U\sqrt{\frac{1}{X_L{}^2} + \frac{1}{R^2}} = \frac{U}{Z}$$

Impedance:

$$\frac{1}{Z} = \sqrt{\frac{1}{X_L{}^2} + \frac{1}{R^2}} = \sqrt{\left(\frac{1}{\omega L}\right)^2 + \frac{1}{R^2}}$$

SUMMARY OF FORMULAE

	D.C.	**A.C.**
In series	$R = R_1 + R_2$	$Z = \sqrt{R^2 + X^2}$
In parallel	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{Z} = \sqrt{\frac{1}{X^2} + \frac{1}{R^2}}$

EXAMPLE

D.C.

$R_1 = 30\ \Omega$ $R_2 = 60\ \Omega$

in parallel

$R = 20\ \Omega$

(33% less than R_1)

A.C.

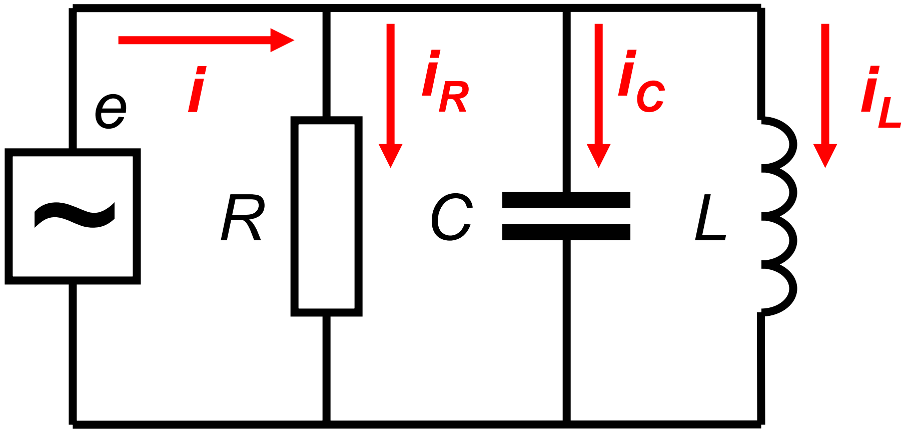
$R = 30\ \Omega$ $X = 60\ \Omega$

in parallel

$Z = 26.8\ \Omega$

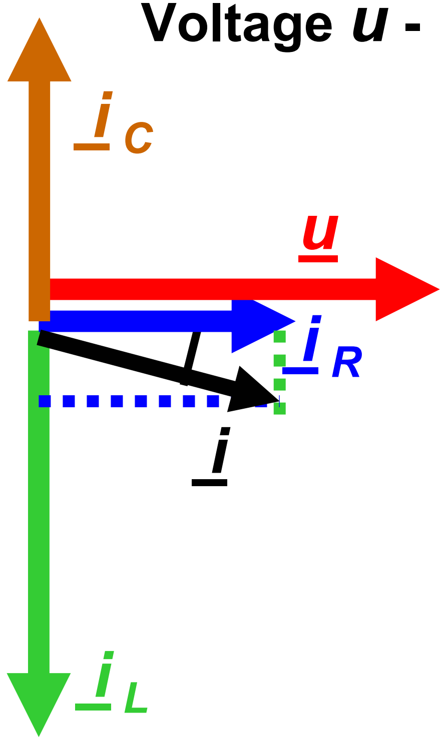
(10% less than R)

EXAMPLE: *LCR* CIRCUIT IN PARALLEL



$\underline{i} = \underline{i}_R + \underline{i}_C + \underline{i}_L$

Voltage *u* - reference



$$I_R = U/R\ , \quad I_C = U/X_C\ , \quad I_L = U/X_L$$

i_R in-phase with u ,

i_L lags u by $\pi/2$, i_C leads u by $\pi/2$

Phase

$$\varphi = \tan^{-1}\frac{I_C - I_L}{I_R} = \tan^{-1}\left[R\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right]$$

Amplitude:

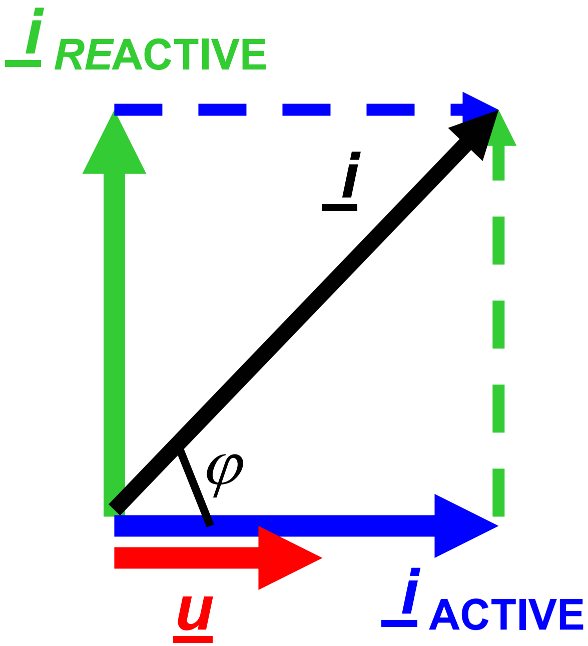
$$I = \sqrt{(I_C - I_L)^2 + I_R{}^2} = U\sqrt{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2 + \frac{1}{R^2}} = \frac{U}{Z}$$

I_L and I_C **may be much larger** than the total current I

Impedance:

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2 + \frac{1}{R^2}} = \sqrt{\left(\omega C - \frac{1}{\omega L}\right)^2 + \frac{1}{R^2}}$$

ACTIVE AND REACTIVE CURRENTS



Current in **any** circuit can be represented

as a sum of two components:

1) in-phase; 2) at phase difference of 90^0 to voltage

In-phase - **active**, or **power** component

$$I_{\text{active}} = I\cos\varphi$$

At 90^0 to voltage - **reactive**, or **quadrature** component

$$I_{\text{reactive}} = I\sin\varphi$$

$$I^2 = (I_{\text{active}})^2 + (I_{\text{reactive}})^2$$

RELATION TO ACTIVE POWER: $P_{\text{active}} = IU\cos\varphi = I_{\text{active}}U$

POWER FACTOR $= \cos\varphi = \dfrac{I_{\text{active}}}{\sqrt{(I_{\text{active}})^2 + (I_{\text{reactive}})^2}}$

Resistors are **active**, while capacitors and coils are **reactive** circuit elements

For purely reactive (capacitive or inductive) circuits, power factor = 0

ADMITTANCE, CONDUCTANCE AND SUSCEPTANCE

For D.C. circuits: R - resistance, $G=1/R$ - **D.C. conductance**

For two resistors in parallel: $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \quad \therefore \quad G=G_1+G_2$$

For A.C. circuits, an equivalent quantity is called **admittance** Y

$$\text{Admittance}=\frac{1}{\text{Impedance}} \;:\; Y=\frac{1}{Z}=\frac{I}{U}$$ Units - siemens [S]

A.C. conductance G is an **active component** of admittance

A **reactive component** of admittance is called **susceptance** (notation - B)

$$G=\frac{I_{\text{active}}}{U} \qquad B=\frac{I_{\text{reactive}}}{U} \qquad \therefore \qquad Y=\sqrt{G^2+B^2}$$

NOTE

G, B and Y may be conductance, susceptance and admittance, respectively, of circuit elements or of the **whole circuit**

ADMITTANCE, CONDUCTANCE AND SUSCEPTANCE IMPEDANCE, RESISTANCE AND REACTANCE: THEIR RELATION

$$G = \frac{I_{\text{active}}}{U} = \frac{I_{\text{active}}}{I}\frac{I}{U} = \frac{R}{Z}\cdot\frac{1}{Z} = \frac{R}{Z^2}$$

$$B = \frac{I_{\text{reactive}}}{U} = \frac{I_{\text{reactive}}}{I}\frac{I}{U} = \frac{X}{Z}\cdot\frac{1}{Z} = \frac{X}{Z^2}$$

Note that $G=1/R$ **only for purely resistive circuits**

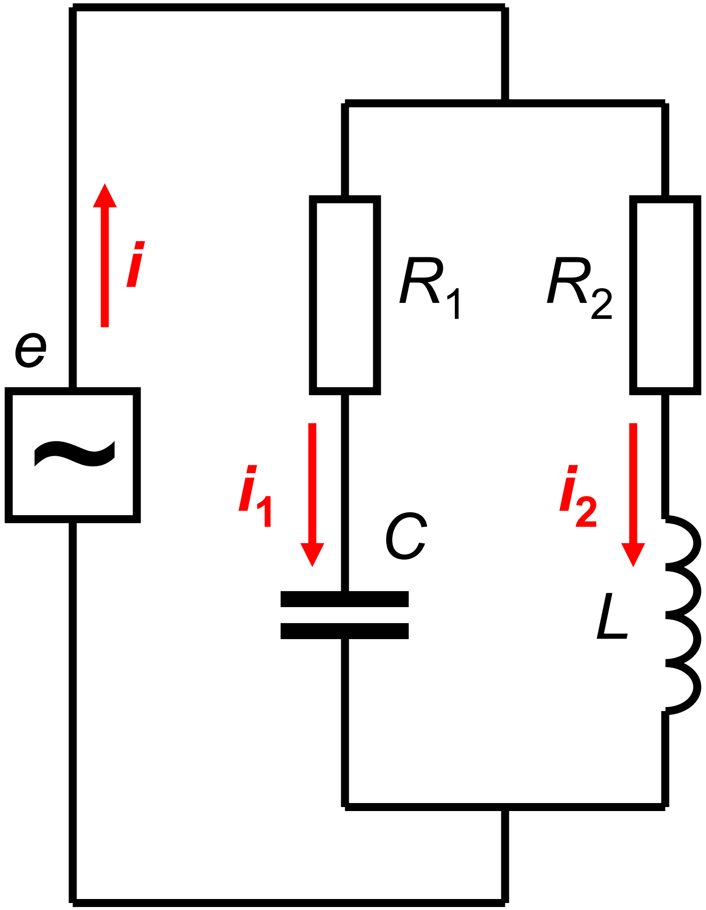
EXAMPLES

If a resistor R_0 is connected **in series** with a reactive element X, resistance of the circuit R is equal to R_0: $R = R_0$

However, if a resistor R_0 is connected **in parallel** with a reactive element X, resistance of the circuit R is **not** equal to R_0: $R \neq R_0$

In this case, **conductance of the circuit** G is equal to conductance of the resistor G_0: $G = G_0 = 1/R_0$

EXAMPLE: MORE COMPLICATED CIRCUIT

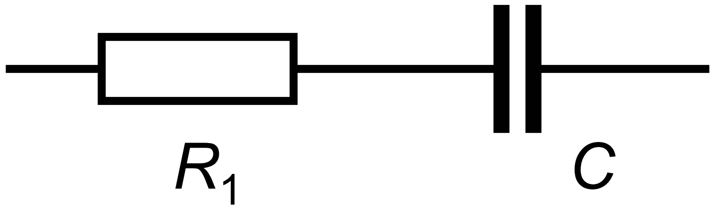


Known parameters: e, frequency, R_1, R_2, L, C.

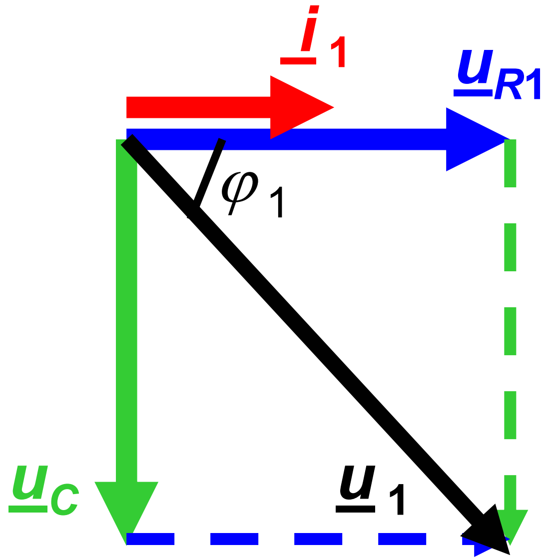
Determine: current, phase angle

STEP 1

Analysis of the R_1C segment

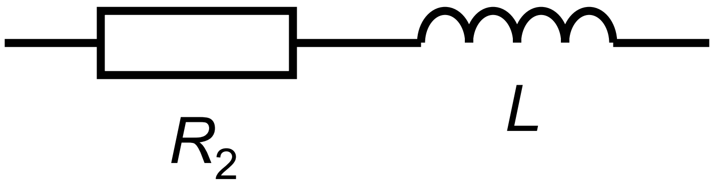


Reference - current i_1

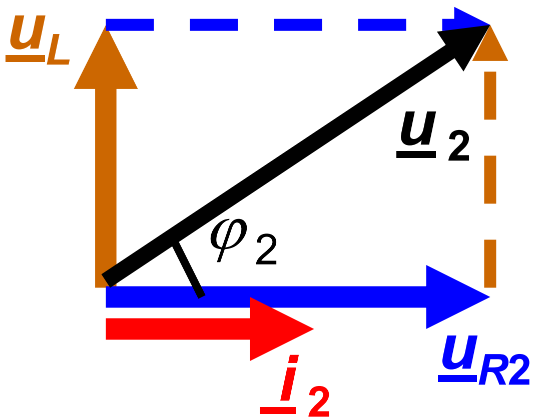


STEP 2

Analysis of the R_2L segment



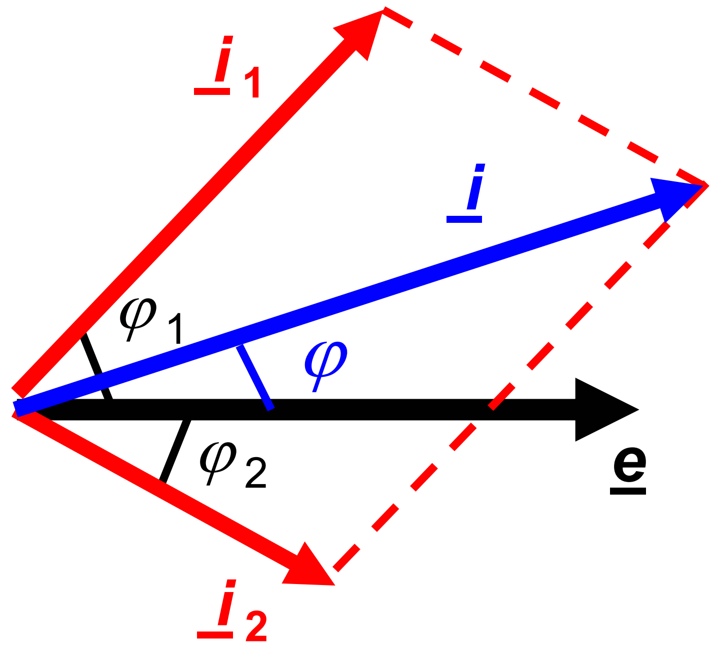
Reference - current i_2



STEP 3

Analysis of the whole circuit

Reference - voltage $u_1 = u_2 = e$



For more or less complicated circuits, the method of phasor diagrams becomes rather cumbersome

Anything more efficient?

Method of complex notations (next lecture)