9. GENERAL NETWORK THEOREMS

Main things to learn

- Superposition theorem
- Thévenin's theorem
- Application of the theorems to d.c. and a.c. circuits

The Superposition theorem and the Thévenin's theorem provide an **alternative approach** to analysis of complicated circuits, in particular, circuits with two or more power sources.

These theorems **may be applied both to d.c. and a.c. circuits**
For a.c. circuits, **complex notations should be used**.
A circuit must be linear!

If there is a circuit which contains several power sources and we analyse the circuit basing on Kirchhoff's laws, we need take the power sources into account **simultaneously**.

Is it possible to consider them separately?

A circuit which contains one or more power sources is called an **active** circuit, or an **active network**

SUPERPOSITION THEOREM

IF

there is a circuit which contains two or more power sources

(an active network)

THEN

we can find the current in any branch or the potential difference across any branch in the following way:

1

We consider the **effect of the first source**, replacing all other sources by resistances equal to their internal resistances

2

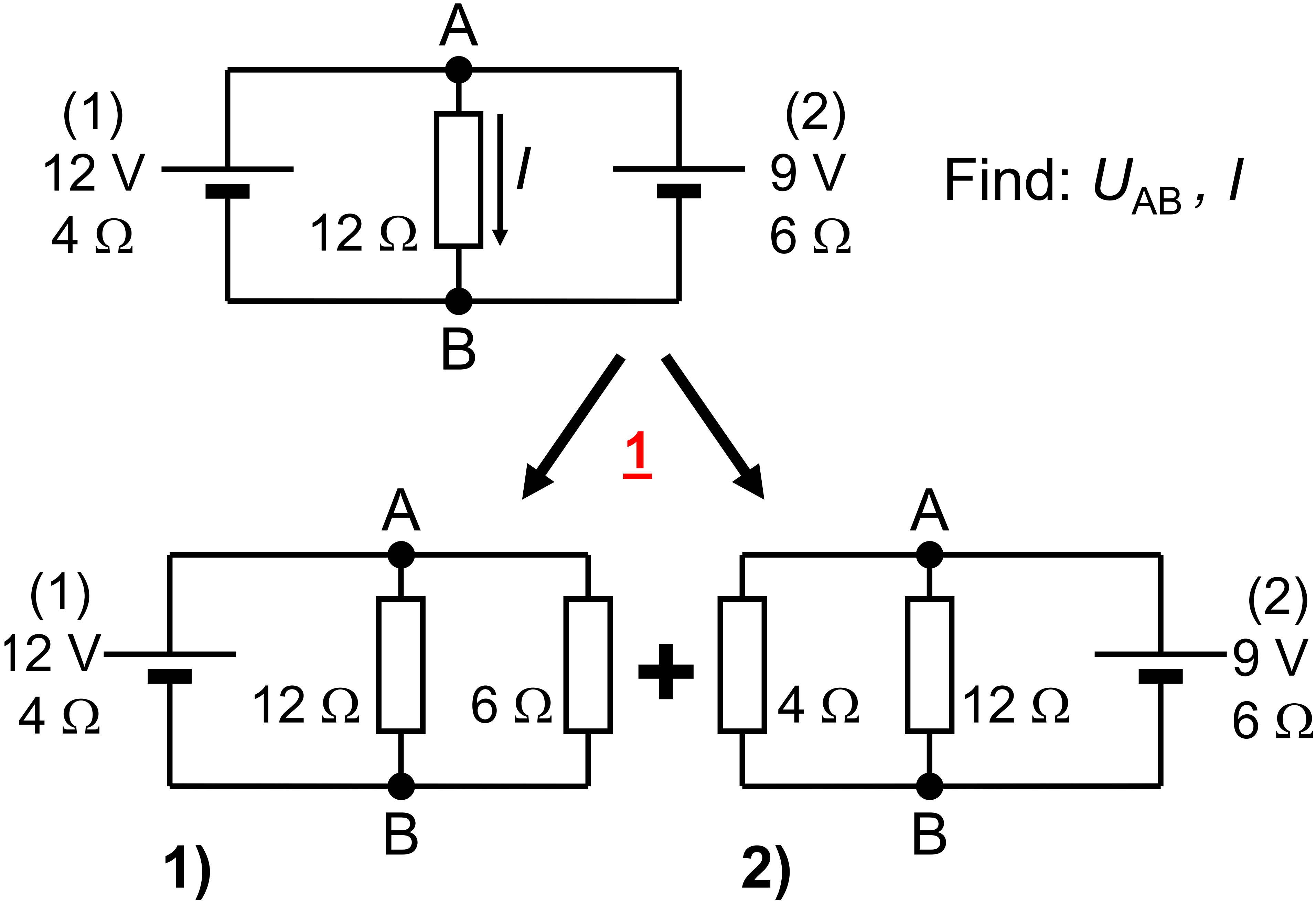
We **successively** consider the effect of each other source in the same way

3

We add their effects

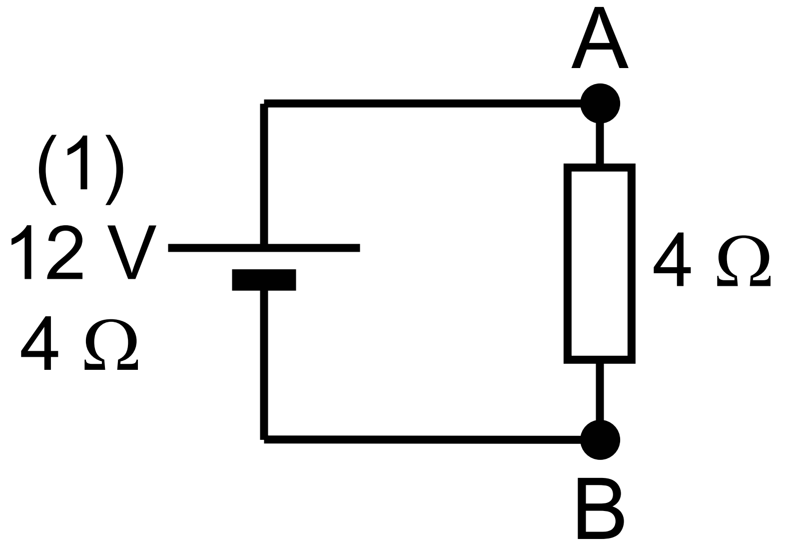
EXAMPLE: D.C. CIRCUIT

Find: U_{AB} , I



2

Circuit 1) is equivalent to

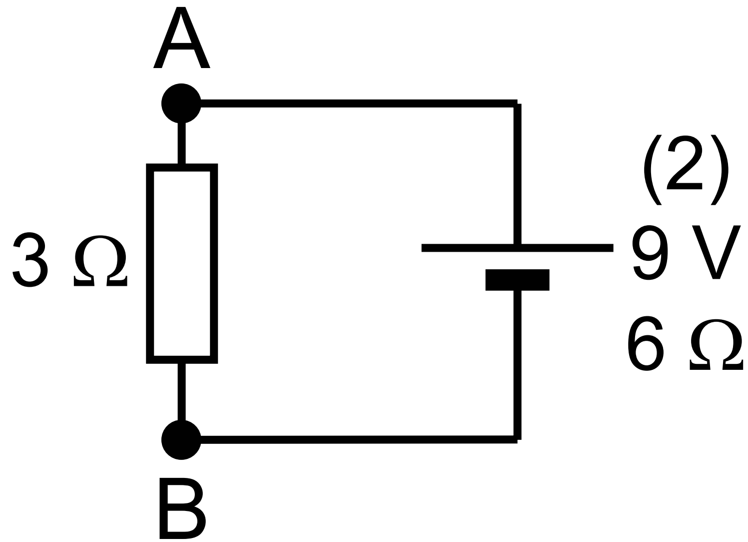


4 Ω is the resistance of the load
as seen by source (1)

$$I_{(1)} = \frac{12\ \text{V}}{(4+4)\ \Omega} = 1.5\ \text{A}$$

$$U_{AB(1)} = 1.5\ \text{A} \times 4\ \Omega = 6\ \text{V}$$

Circuit 2) is equivalent to



3 Ω is the resistance of the load
as seen by source (2)

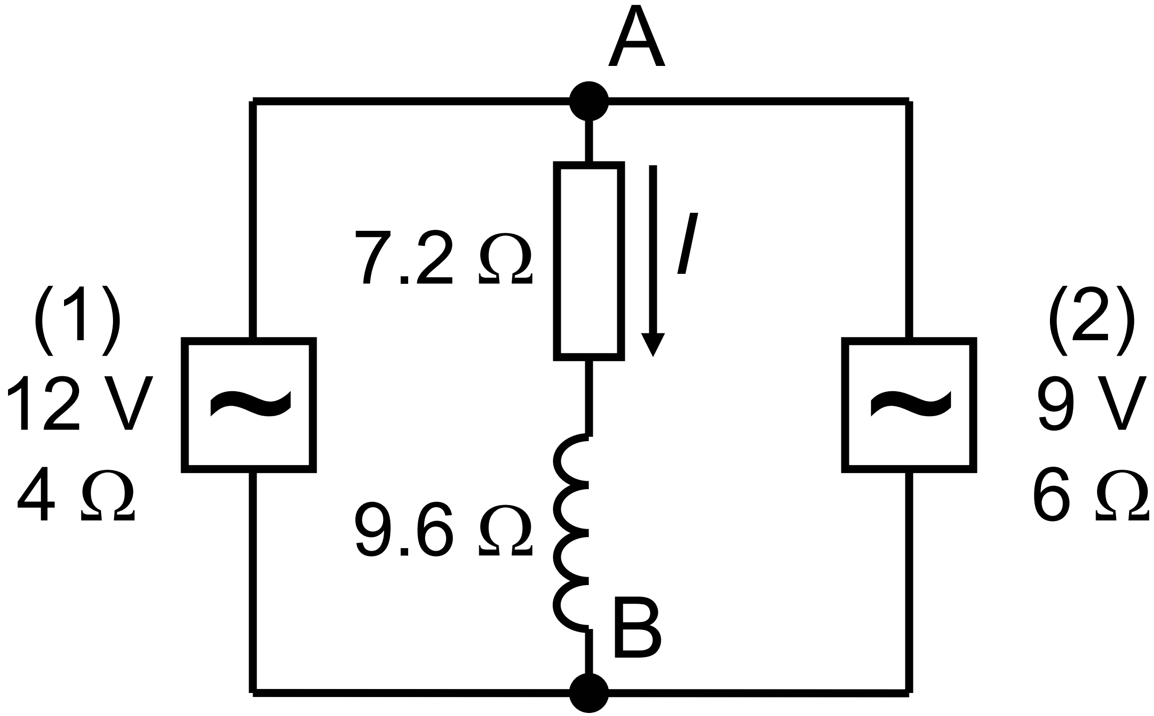
$$I_{(2)} = \frac{9\ \text{V}}{(6+3)\ \Omega} = 1\ \text{A}$$

$$U_{AB(2)} = 1\ \text{A} \times 3\ \Omega = 3\ \text{V}$$

3

$$U_{AB} = U_{AB(1)} + U_{AB(2)} = 6\ \text{V} + 3\ \text{V} = 9\ \text{V} \quad ; \quad I = \frac{9\ \text{V}}{12\ \Omega} = 0.75\ \text{A}$$

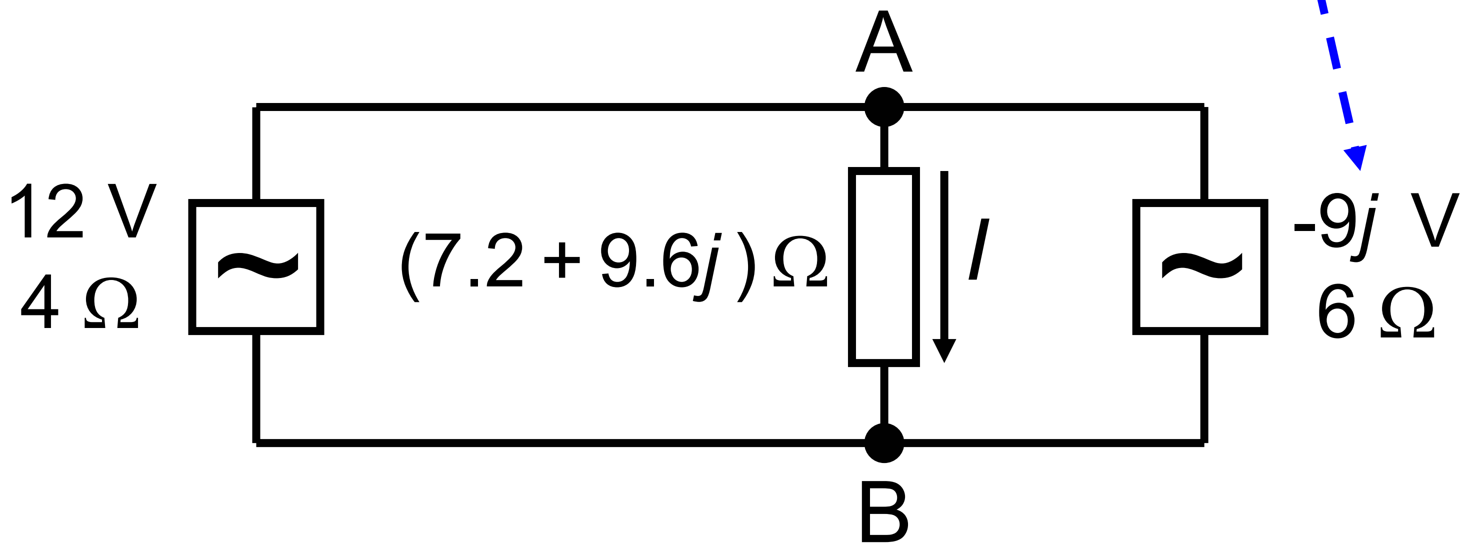
EXAMPLE: A.C. CIRCUIT



Find: U_{AB} , I

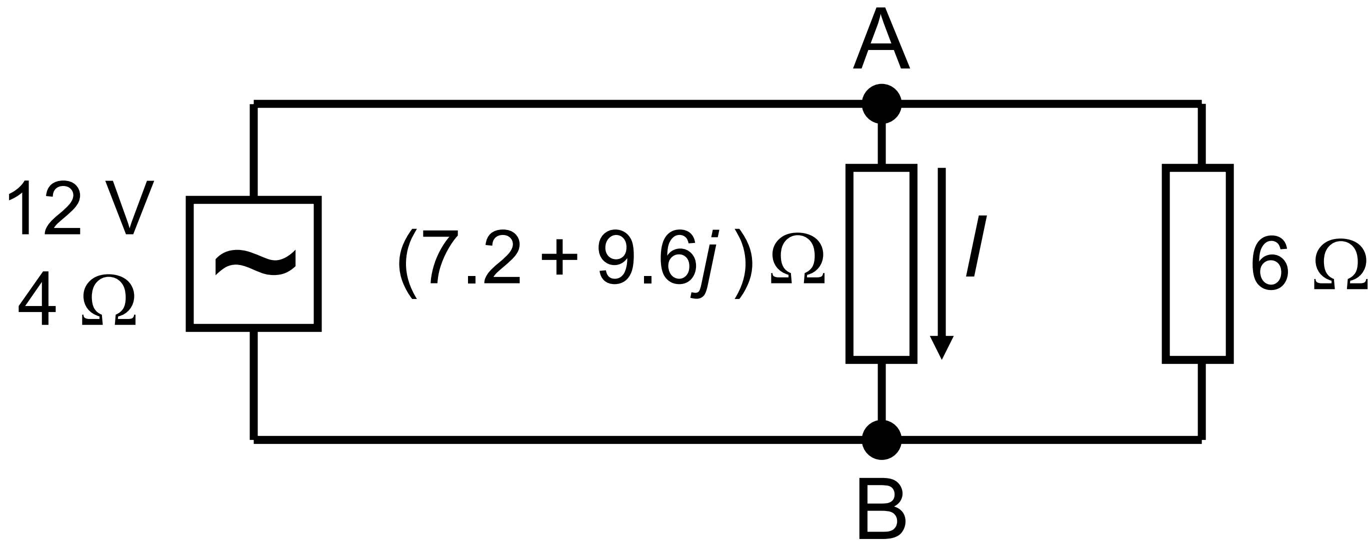
Assume that source (2) lags source (1) by 90^0

Using complex notations:



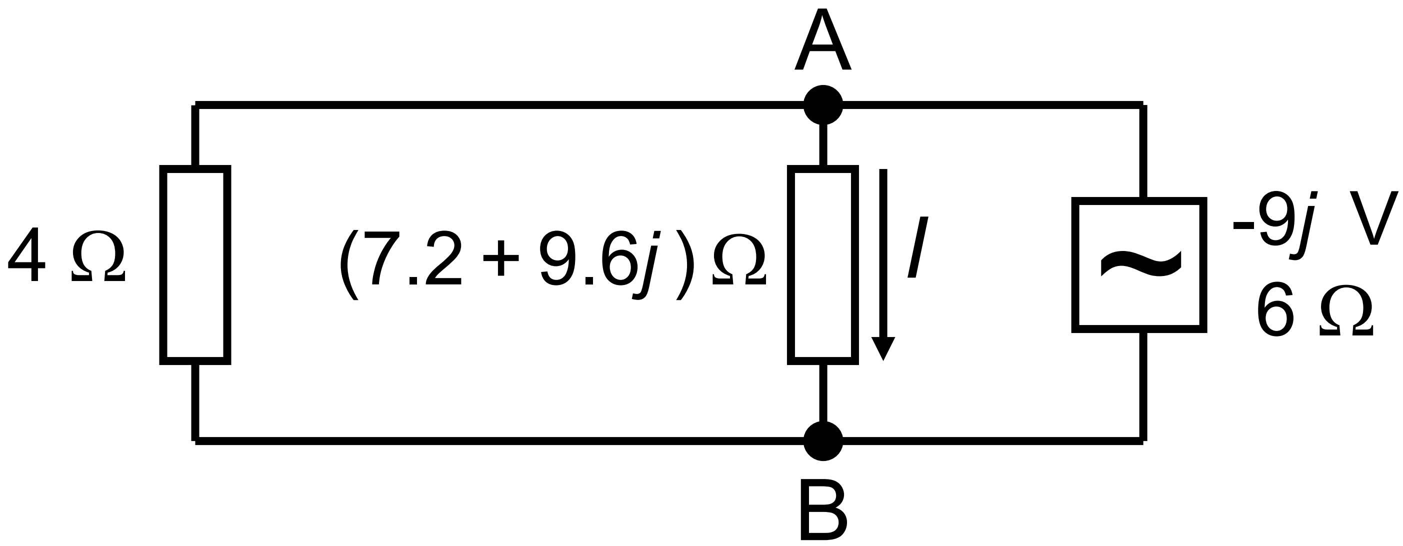
1 THIS IS EQUIVALENT TO

1)



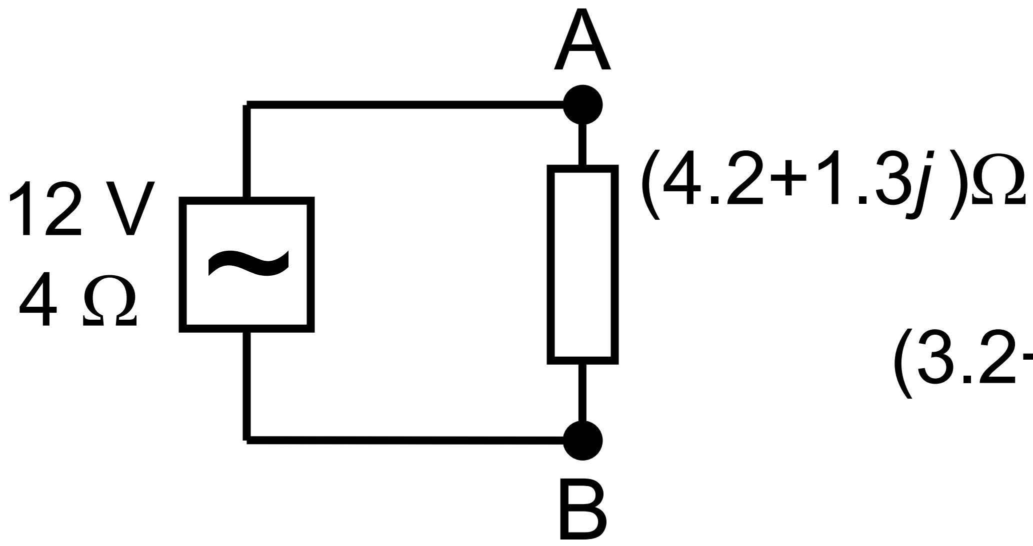
+

2)

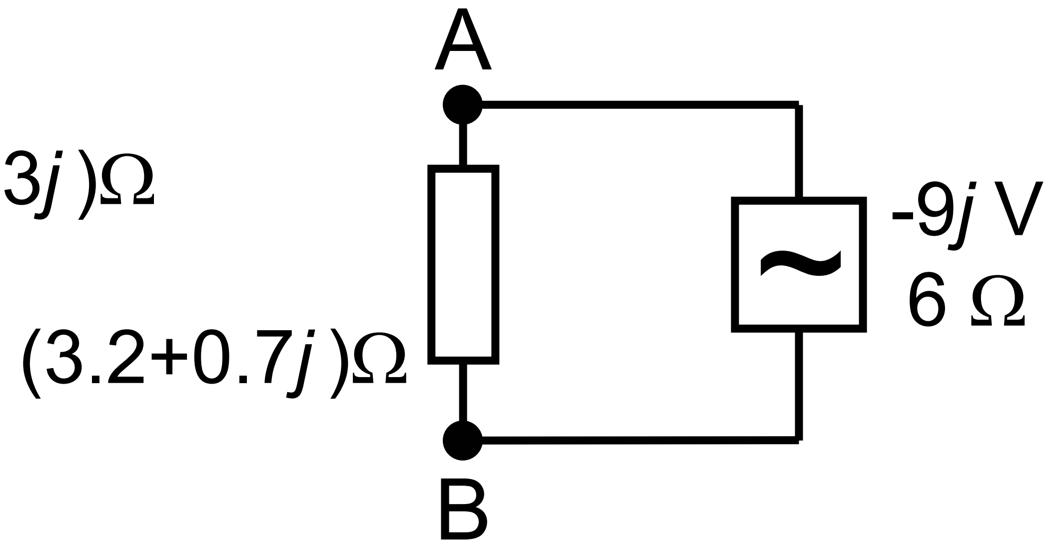


2

Circuit 1) is equivalent to



Circuit 2) is equivalent to



$$I_{(1)} = \frac{12\ \text{V}}{[4 + (4.2 + 1.3j)]\ \Omega} = (1.4 - 0.2j)\ \text{A}$$

$$U_{\text{AB}(1)} = (1.4 - 0.2j)\ \text{A} \times (4.2 + 1.3j)\ \Omega = (6.1 + j)\ \text{V}$$

$$I_{(2)} = \frac{-9j\ \text{V}}{[6 + (3.2 + 0.7j)]\ \Omega} = (-0.1 - j)\ \text{A}$$

$$U_{\text{AB}(2)} = (-0.1 - j)\ \text{A} \times (3.2 + 0.7j)\ \Omega = (0.4 - 3.3j)\ \text{V}$$

3 FINAL RESULT

$$U_{AB} = U_{AB(1)} + U_{AB(2)} = (6.1 + j)\,\text{V} + (0.4 - 3.3j)\,\text{V} = (6.5 - 2.3j)\,\text{V}$$

$$I = \frac{(6.5 - 2.3j)\,\text{V}}{(7.2 + 9.6j)\ \Omega} = (0.17 - 0.55j)\,\text{A}$$

Amplitude

$$|U_{AB}| = |6.5 - 2.3j| = \sqrt{6.5^2 + 2.3^2} = 6.9\ \text{V}$$

$$|I| = |0.2 - 0.5j| = \sqrt{0.17^2 + 0.55^2} = 0.57\ \text{A}$$

Phase

If the phase of the source (1) is taken as reference (zero), phases are:

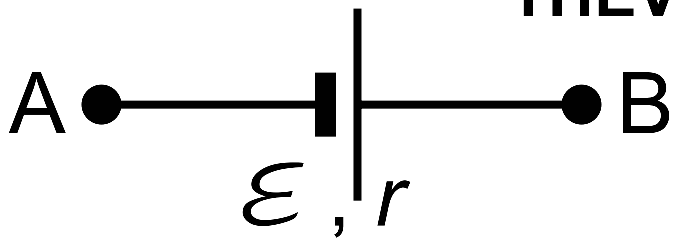
for U_{AB} : $\varphi = \tan^{-1}(-2.3 / 6.5) = -19^0$

(voltage U_{AB} lags the voltage from the source (1) by 19^0)

for I : $\varphi = \tan^{-1}(-0.55 / 0.17) = -73^0$

(current I lags the voltage from the source (1) by 73^0)

THÉVENIN'S THEOREM



Any battery can be characterised by two parameters:

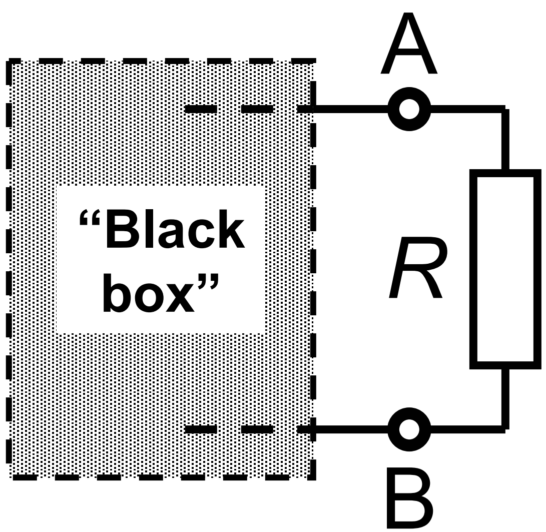
1) EMF $\mathcal{E}$; 2) internal resistance r

The EMF $\mathcal{E}$ equals the open-circuit voltage between the terminals A and B;

r equals the ratio of $\mathcal{E}$ to the current if the terminals A and B are short-circuited

Thévenin's theorem

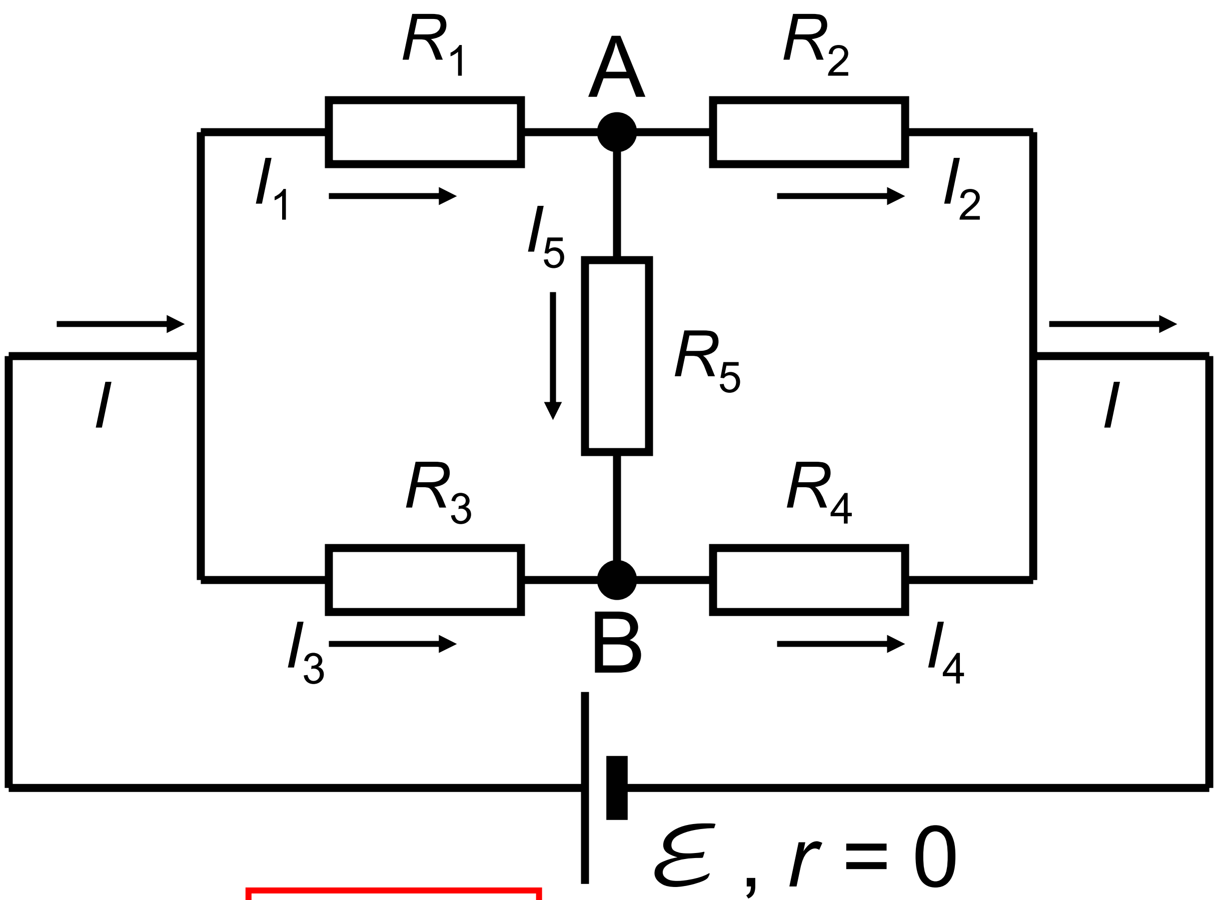
Any active circuit (network) having terminals A and B is equivalent and can be replaced by a constant-voltage source which has an EMF $\mathcal{E}$ and an internal resistance *r*.



Another formulation:

The current through a resistor R connected across any two points A and B of an active network can be obtained by dividing the potential difference between A and B, with R disconnected, by (R + r) where r is the resistance of the network measured between points A and B with R disconnected and the sources of EMF replaced by their internal resistances.

EXAMPLE



$\mathcal{E} = 24$ V

$R_1 = 14\ \Omega$

$R_2 = 42\ \Omega$

$R_3 = 48\ \Omega$

$R_4 = 24\ \Omega$

$R_5 = 13.5\ \Omega$

Find: U_{AB} , I_5

$$U_1 = I_1 R_1$$
$$U_2 = I_2 R_2$$
$$U_3 = I_3 R_3$$
$$U_4 = I_4 R_4$$
$$U_5 = I_5 R_5$$

$$-I_1 + I_2 + I_5 = 0$$
$$-I_3 + I_4 - I_5 = 0$$
$$I = I_1 + I_3$$
$$R = \frac{\mathcal{E}}{I}$$

$$U_1 + U_2 = \mathcal{E}$$
$$U_3 + U_4 = \mathcal{E}$$
$$U_1 + U_5 + U_4 = \mathcal{E}$$

USING THE THÉVENIN'S THEOREM

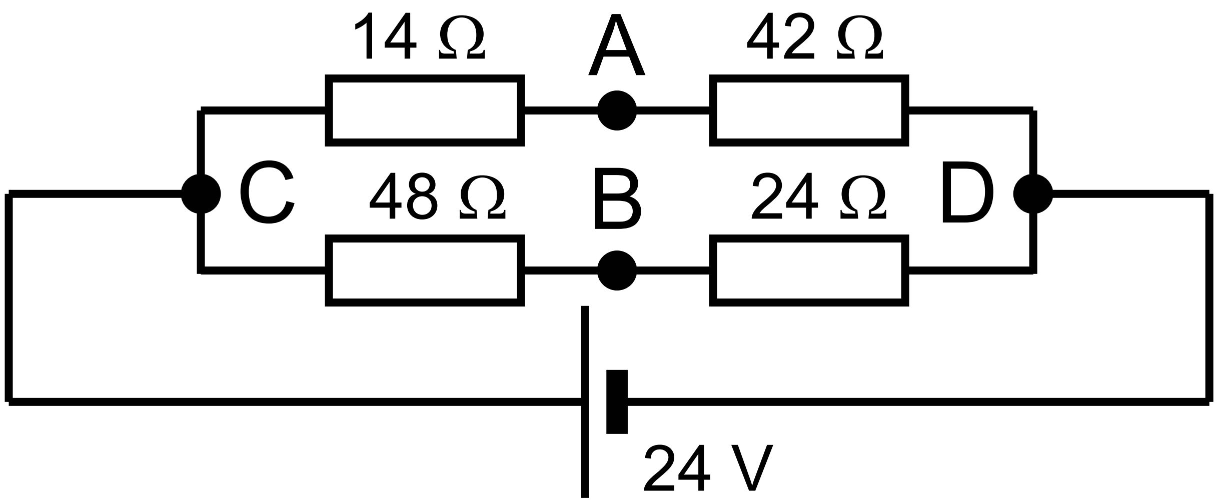
1

Determine U^*_{AB} when R_5 is disconnected

$U_{CA} = 6$ V ; $U_{AD} = 18$ V

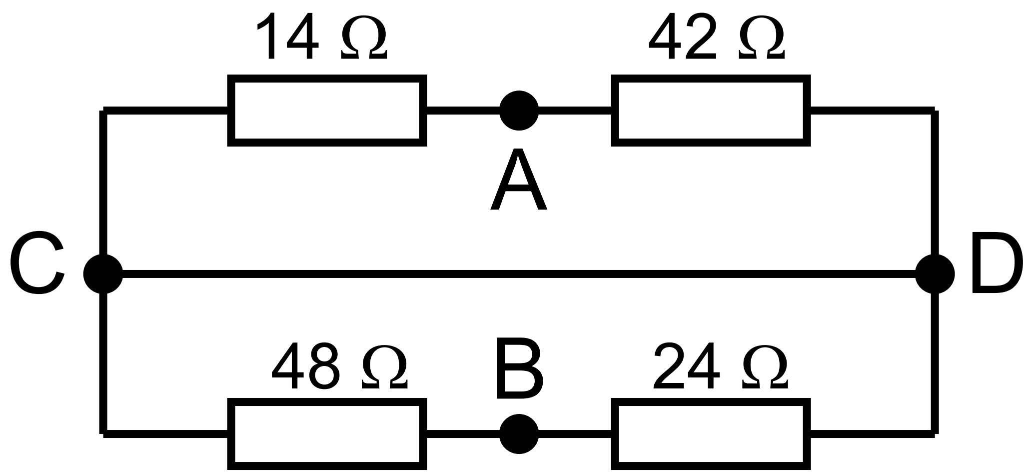
$U_{CB} = 16$ V ; $U_{BD} = 8$ V

$U^*_{AB} = -U_{CA} + U_{CB} = 10$ V



2

Determine r^*_{AB} when R_5 is disconnected and the battery is replaced by equivalent (zero) resistance



$$\frac{1}{R_{AC}} = \frac{1}{14\,\Omega} + \frac{1}{42\,\Omega} \quad \therefore \quad R_{AC} = 10.5\,\Omega$$

$$\frac{1}{R_{CB}} = \frac{1}{48\,\Omega} + \frac{1}{24\,\Omega} \quad \therefore \quad R_{CB} = 16\,\Omega$$

$$\therefore \quad r^*_{AB} = 10.5 + 16 = 26.5\,\Omega$$

FINAL RESULT

3

Determine current I_5 which flows through R_5

$$I_5 = \frac{U^*_{AB}}{R_5 + r^*_{AB}} = \frac{10\ \text{V}}{13.5\ \Omega + 26.5\ \Omega} = \frac{10\ \text{V}}{40\ \Omega} = 0.25\ \text{A}$$

Determine voltage U_{AB}

$$U_{AB} = I_5 \times R_5 = 0.25\ \text{A} \times 13.5\ \Omega = 3.4\ \text{V}$$

In the same way the Thévenin's theorem can be applied to a.c. circuits.

Complex notations should be used.

<u>The circuit must be linear !!!</u>